[2]Lin, K. C., and Schab, D., "The Performance Assessment of the Dead Reckoning Algorithms in DIS," *Simulation*, Vol. 63, No. 5, 1994, pp. 318–325.

[3]Lin, K. C., "Dead Reckoning and Distributed Interactive Simulation," *Proceedings of SPIE's International Symposium on Aerospace, CR58* (Orlando, FL), SPIE Optical Engineering Press, Bellingham, WA, 1995, pp. 16–36.

[4]Lin, K. C., and Schab, D., "Network Load in Distributed Interactive Simulation," *Journal of Aircraft*, Vol. 32, No. 6, 1995, pp. 1392, 1393.

[5]Lin, K. C., Wang, M., Wang, J., and Schab, D., "Rating of Image Smoothness in Distributed Interactive Simulation," *International Journal in Computer Simulation* (submitted for publication).

[6]Agresti, A., *Categorical Data Analysis*, Wiley, New York, 1990.

Simple Prediction Method for Location of Vortex Breakdown on Delta Wings

Lance W. Traub*
Texas A&M University, College Station, Texas 77843

Nomenclature

c = chord
g = similarity function
K = Sychev similarity parameter
U_∞ = freestream velocity
x = chordwise direction
α = angle of attack
Γ = vortex circulation
$\Gamma_{\alpha>BD\text{-}TE}$ = circulation at trailing edge at angles of attack beyond that at which breakdown occurs
ε = wing apex half-angle
Λ = leading-edge sweep angle

Subscripts
BD = breakdown
BD-TE = breakdown at trailing edge
r = root

Introduction

BEYOND moderate angles of attack the flowfield over slender delta wings is usually characterized by the formation of leading-edge vortices. These vortices form and are strengthened by vorticity emanating from the wing's leading edge. However, as the angle of attack of the wing increases, the leading-edge vortices may break down, a phenomena associated with a marked deceleration of the vortex core, and increase in the diameter of the resulting structure. Unwelcome characteristics of vortex breakdown (e.g., a flattening of the lift curve and a destabilizing nose up pitching moment, etc.) have resulted in numerous studies of the phenomena.[1–3] Generally, attempts to find a universal scaling parameter to correlate breakdown have not been satisfactory.[4] However, studies have suggested that breakdown is associated with the concentration and distribution of vorticity, and the ability of the vortex to convect it axially downstream.[5] The apparent success of the Rossby number[6] (i.e., the ratio of the axial velocity to the maximum rotary velocity at the edge of the viscous core), as a criterion for the prediction of the onset of breakdown, also supports the concept of a threshold level for the convection of axial momentum or vorticity downstream.

Although numerical predictions of the onset of vortex breakdown are feasible, it is still useful to have simple analytic expressions for predicting the location of the breakdown,

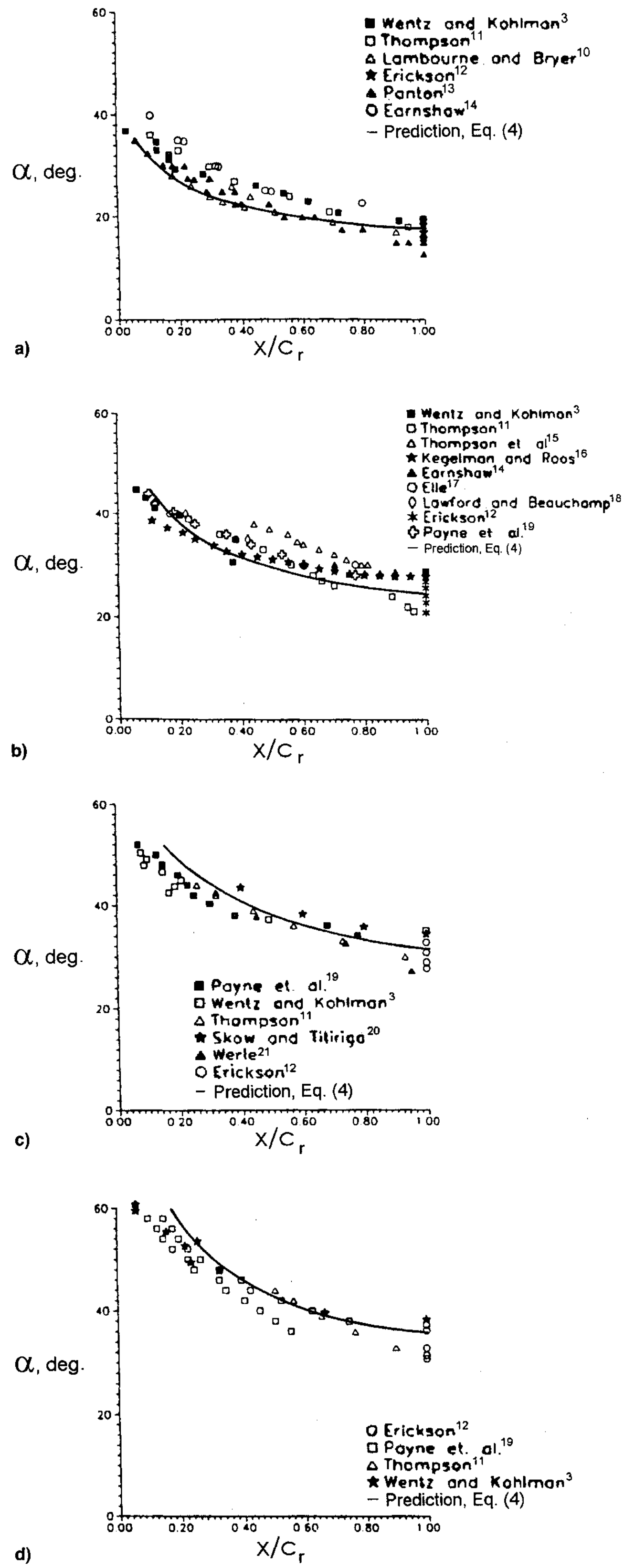

Fig. 1 Location of vortex breakdown position for various leading-edge sweep angles. Λ = a) 65, b) 70, c) 75, and d) 80 deg.

Received Oct. 18, 1995; revision received Nov. 22, 1995; accepted for publication Nov. 27, 1995.

*Graduate Student, Aerospace Engineering Department.

where extensive numerical solutions are not practical. In this Note, expressions are derived to predict the chordwise variation of breakdown with incidence for delta wings.

Discussion of Method

Hemsch and Luckring[7] showed that by using Sychev similarity parameters, the vortex strength at the trailing edge of slender wings could be correlated. The parameter was given in the form (where the exponent of K equal to 1.2 was shown to correlate the numerical data of Smith[7]):

$$g = AK^{1.2} = \Gamma/U_\infty c_r \tan^2\varepsilon \cos\alpha \tag{1}$$

where K is given by

$$K = \tan\alpha/\tan\varepsilon$$

Visser and Nelson[5] showed that setting $A = 4.63$ yielded an effective correlation of their test data and that of other researchers, when g was plotted as a function of K. Recasting Eq. (1) in terms of nondimensional circulation gives

$$\Gamma/U_\infty c_r = 4.63 \tan^{0.8}\varepsilon \tan^{1.2}\alpha \cos\alpha \tag{2}$$

Criteria to correlate breakdown consisting of a fixed value of circulation as suggested by Jumper et al.[8] have generally not been successful.[4] However, using the large base of experimental data contained in Ref. 4 showed that setting $\Gamma/U_\infty c_r = 0.68$ and solving for α using Eq. (2) yielded values within the experimental scatter of the data for the prediction of $\alpha_{\text{BD-TE}}$ for leading-edge sweep angles of 65, 70, 75, and 80 deg. However, the predictions for a given data set were not found to be particularly consistent. Using Eq. (1) and K to correlate the experimental results in Fig. 1 (based upon that in Ref. 4), allows an exponential curve to be fitted as given by Eq. (3), which is more consistent in predicting $\alpha_{\text{BD-TE}}$:

$$\tan\alpha_{\text{BD-TE}} = 13.47 \tan\varepsilon\, e^{-6.9\varepsilon} \tag{3}$$

Having determined an expression to estimate $\alpha_{\text{BD-TE}}$, it is necessary to establish the location of breakdown with increasing incidence when it has progressed forward of the trailing edge. The form of a suitable expression for $(x/c_r)_{\text{BD}}$ was determined by initially assuming that at breakdown (for $\alpha > \alpha_{\text{BD-TE}}$), the nondimensional circulation ($\Gamma_{\text{BD}}/U_\infty c_r$) is constant, and equal to the value when breakdown first occurs at the trailing edge, i.e., $\Gamma_{\text{BD}}/U_\infty c_r = \Gamma_{\text{BD-TE}}/U_\infty c_r$, as determined using Eqs. (2) and (3). Slender wing theory,[9] using assumptions of conical flow, suggests that the strength of the leading-edge vortex increases linearly with distance from the apex; a result that has also been seen in experiment.[5] Thus it was also presumed that the flow is conical such that circulation increases linearly from the apex.

Interpolating between the value of circulation at the trailing-edge $\Gamma_{\alpha>\text{BD-TE}}/U_\infty c_r$, found using Eq. (2) for a given α (noting that circulation at the apex is zero) to locate the position where the circulation is equal to $\Gamma_{\text{BD-TE}}/U_\infty c_r$, yields an expression of the form:

$$\left(\frac{x}{c_r}\right)_{\text{BD}} = \frac{\Gamma_{\text{BD-TE}}}{\Gamma_{\alpha>\text{BD-TE}}}\left(\frac{x_{\text{TE}}}{c_r}\right) = \frac{\Gamma_{\text{BD-TE}}}{\Gamma_{\alpha>\text{BD-TE}}}$$

The value of circulation at breakdown ($\alpha > \alpha_{\text{BD-TE}}$) with increasing incidence generally is not constant, and it reduces, approaching a value of zero as breakdown approaches the leading edge. The expression mentioned previously needs to be modified to take into account this reduction. A suitable modification was determined to be of the form given by Eq. (4):

$$\left(\frac{x}{c_r}\right)_{\text{BD}} = \left(\frac{\Gamma_{\text{BD-TE}}}{\Gamma_{\alpha>\text{BD-TE}}}\right)^n \tag{4}$$

Estimation of the progression of breakdown for a specific planform would consist of solving Eq. (3) for $\alpha_{\text{BD-TE}}$. Next, the corresponding circulation at the trailing edge for $\alpha_{\text{BD-TE}}$ would be determined using Eq. (2). Subsequently, the chordwise variation of breakdown with increasing incidence may be determined using Eq. (4).

Setting the index $n = 3$ yielded the predictions shown in Fig. 1. It may be seen in the figure that Eq. (4) shows good correlation with the experimental data. Although the scatter in the measurements is fairly substantial, all of the results show essentially similar trends for the variation of breakdown location with incidence. It is evident that the general form of the plots is adequately represented by Eq. (4). The figure also clearly shows that increasing wing slenderness is accompanied by an increase in the angle of attack at which breakdown first occurs at the wing's trailing edge. It is notable that the angle-of-attack range, required for breakdown to progress from the wing's trailing edge to the apex, is seen to be roughly constant for the featured planforms, and spans approximately 20–25 deg.

Summary

Expressions were derived to predict the variation of vortex breakdown location with incidence for delta wings. The method uses a Sychev similarity parameter to estimate the vortex circulation at the trailing edge. An interpolation equation, determined on the basis of conical flow and modified to take into account breakdown effects, was derived to estimate the breakdown position. The method was compared with experimental data, and good agreement was shown.

References

[1]Leibovich, S., "Vortex Stability and Breakdown: Survey and Extension," *AIAA Journal*, Vol. 22, No. 9, 1984, pp. 1192–1206.

[2]Brown, G. L., and Lopez, J. M., "Axisymmetric Vortex Breakdown, Part 2. Physical Mechanisms," *Journal of Fluid Mechanics*, Vol. 221, 1990, pp. 553–576.

[3]Wentz, W. H., and Kohlman, D. L., "Vortex Breakdown on Slender Sharp Edged Wings," *Journal of Aircraft*, Vol. 8, No. 3, 1971, pp. 156–161.

[4]Gursul, I., "Criteria for Location of Vortex Breakdown over Delta Wings," *Aeronautical Journal*, Vol. 99, No. 985, 1995, pp. 194–196.

[5]Visser, K. D., and Nelson, R. C., "Measurements of Circulation and Vorticity in the Leading-Edge Vortex of a Delta Wing," *AIAA Journal*, Vol. 31, No. 1, 1993, pp. 104–111.

[6]Robinson, B. A., Barnett, R. M., and Agrawal, S., "Simple Numerical Criterion for Vortex Breakdown," *AIAA Journal*, Vol. 32, No. 1, 1994, pp. 116–122.

[7]Hemsch, M. J., and Luckring, J. M., "Connection Between Leading Edge Sweep, Vortex Lift, and Vortex Strength for Delta Wings," *Journal of Aircraft*, Vol. 27, No. 5, 1990, pp. 473–475.

[8]Jumper, E. J., Nelson, R. C., and Cheung, K., "A Simple Criterion for Vortex Breakdown," AIAA Paper 93-0866, Jan. 1993.

[9]Smith, J. H. B., "Improved Calculation of Leading Edge Separation from Slender, Thin, Delta Wings," *Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences*, Vol. 306, No. 1484, 1968, pp. 67–90.

[10]Lambourne, N. C., and Bryer, D. W., "The Bursting of Leading Edge Vortices—Some Observation and Discussion of the Phenomenon," Aeronautical Research Council, R&M 3282, April 1962.

[11]Thompson, D. H., "A Water Tunnel Study of Vortex Breakdown over Wings with Highly Swept Leading Edges," Australian Research Labs., ARL/A 356, Melbourne, Australia, May 1975.

[12]Erickson, G. E., "Water-Tunnel Studies of Leading Edge Vortices," *Journal of Aircraft*, Vol. 19, No. 6, 1982, pp. 442–448.

[13]Panton, R. L., "Effects of a Contoured Apex on Vortex Breakdown," *Journal of Aircraft*, Vol. 27, No. 3, 1990, pp. 285–288.

[14]Earnshaw, P. B., "Measurements of Vortex Breakdown Position at Low Speed on a Series of Sharp Edged Symmetrical Models," Royal Aircraft Establishment, RAE TR 64047, Nov. 1964.

[15]Thompson, S. A., Batill, S. M., and Nelson, R. C., "Separated Flowfield on a Slender Wing Undergoing Transient Pitching Motions," *Journal of Aircraft*, Vol. 28, No. 8, 1991, pp. 489–495.

[16]Kegelman, J. T., and Roos, F. W., "Effects of Leading Edge Shape

and Vortex Burst on the Flowfield of a 70-Degree-Sweep Delta Wing,'' AIAA Paper 89-0086, Jan. 1989.

[17]Elle, B. J., ''An Investigation at Low Speed of the Flow near the Apex of Thin Delta Wings with Sharp Leading Edges,'' Aeronautical Research Council, R&M 3176, Jan. 1958.

[18]Lawford, J. A., and Beauchamp, A. R., ''Low Speed Wind Tunnel Measurements on a Thin Sharp Edged Delta Wing with 70° Leading Edge Sweep, with Particular Reference to the Position of Leading Edge Vortex Breakdown,'' Aeronautical Research Council, R&M 3338, Nov. 1961.

[19]Payne, F. M., Ng, T. T., Nelson, R. C., and Schiff, L. B., ''Visualization and Wake Surveys of Vortical Flow over a Delta Wing,'' *AIAA Journal*, Vol. 26, No. 1, 1988, pp. 137–143.

[20]Skow, A. M., and Titiriga, A., ''A Survey of Analytical and Experimental Techniques to Predict Aircraft Dynamic Characteristics at High Angles of Attack,'' *Dynamic Stability Parameter*, CP-235, AGARD, 1978 (Paper 19).

[21]Werle, H., ''Sur l'eclatement des Tourbillons d'apex d'une Aile Delta aux Fibles Vitesses,'' *La Recherche Aeronautique*, Vol. 74, Jan.–Feb. 1960, pp. 23–30.